CHAPTER 3: Applications of the Derivative
Concepts/Skills to know:

- Identify **extrema** (maximum and minimum, absolute and local) of a continuous function ***f(x)*** on an interval.
- Know the **Extreme Value Theorem**
  If ***f(x)*** is continuous on a closed interval [a, b] then we are guaranteed absolute extrema,
  an AbsMax value and an AbsMin value, on [a, b].
- Define critical numbers of ***f(x)*** as the value of ***c*** in the domain of ***f(x)*** where ***f'(c)***=0 *or* ***f'(c)*** does not exist.
- Know that critical numbers **CNs** are the <u>only</u> places where local extrema (LMax or LMin) <u>can</u> appear but <u>not all</u> critical numbers are extrema.
- Find extrema of a continuous function ***f(x)*** on a closed interval [a, b] by doing the following:
  find all the critical numbers of ***f(x)*** in (a, b)
  evaluate ***f(c)*** at each critical number ***c*** in (a, b)
  evaluate ***f(a)*** and ***f(b)*** at each endpoint of [a, b]
  find the **least** of these values as the **minimum** and the **greatest** of these values as the **maximum**
- Know the **Mean Value Theorem**
  If ***f(x)*** is <u>continuous</u> on the closed interval [a, b] and <u>differentiable</u> on the open interval (a, b),
  then there exists a number ***c*** in (a, b) such that:
  $$f'(c) = \frac{f(b) - f(a)}{b - a}$$
- Use the **Mean Value Theorem** to find the point (*c, f(c)*)on the graph of a function ***f(x)*** at which the tangent line has the same slope as the secant line through 2 other points (*a, f(a)*) and (*b, f(b)*) of the graph.
- Given a position function ***f(t)*** (position as a function of time):
  find ***average velocity*** between two positions and their associated times, ($t_1, f(t_1)$) and ($t_2, f(t_2)$) and
  find the time ***t*** and position ***f(t)*** at which the ***instantaneous velocity*** is equal to this average velocity.
- Know the **First Derivative Test**
  When ***c*** is a critical number for ***f(x)*** and ***f(x)*** is <u>continuous</u> at ***c*** and <u>differentiable</u> on an open interval and **c** is at an open endpoint:
  if ***f'(x)*** changes from positive >0 to negative <0, then ***f(x)*** changes from increasing↑ to decreasing↓
  ∴ ***f(c)*** is a LMax of ***f(x)***
  if ***f'(x)*** changes from negative <0 to positive>0, then ***f(x)*** changes from decreasing↓ to increasing↑
  ∴ ***f(c)*** is a LMin of ***f(x)***
  if ***f'(x)*** does not change in sign, then either ***f(x)*** stays increasing↑ or ***f(x)*** stays decreasing↓
  ∴ ***f(c)*** is not a local extrema of ***f(x)***
- Use **First Derivative Test** to find the closed interval where the function ***f(x)*** increases↑ or decreases↓ and use this information to classify the local extrema of the function ***f(x)*** and sketch graph of ***f(x)***.
- Know the **Second Derivative Test**
  When ***f(x)*** is differentiable on an open interval containing a critical number ***c*** and ***f'(c)***=0:
  if ***f''(c)***<0, then ***f'(x)*** is decreasing↓, **C**oncavity is **D**ownward and ***f(c)*** is a LMax of ***f(x)***
  if ***f''(c)***>0, then ***f'(x)*** is increasing↑, **C**oncavity is **U**pward and ***f(c)*** is a LMin of ***f(x)***
- Identify a **point of inflection** where the concavity of a function switches from upward to downward or from downward to upward.
- Look for points of inflection where ***f''(c)***=0 or ***f''(c)*** does not exist.
  If ***f''(c)*** DNE, then use open intervals and the 2nd Derivative Test to look for concavity switch.
- Use the **Second Derivative Test** to find concavities and their open intervals,
  local extrema and their coordinates, and points of inflection for the function ***f(x)***.
- Find all local extrema of **sine** and **cosine** function for the closed interval [-2π, 2π].